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### New Method to Determine Elastic Constants of Nematic Liquid Crystal From C-V Curve

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## NEW METHOD TO DETERMINE ELASTIC CONSTANTS OF NEMATIC LIQUID CRYSTAL FROM C-V CURVE

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**Abstract** It is found theoretically and experimentally that a linear relationship between capacitance and reciprocal voltage of a nematic liquid crystal cell of homogeneous alignment. From this slope, the ratio of elastic constants  $K_{33}/K_{11}$  can be accurately obtained.

Twisted nematic liquid crystal display devices (TN-LCDs) have been widely used because of their low driving voltage and low power consumption. According to expansion of their usage, their multiplexability are expected to be improved for higher multiplex driving. Several workers<sup>1-3</sup> investigated the relation between multiplexability and material parameters and clarified that ratio of bend and splay elastic constants  $K_{33}/K_{11}$  of nematic liquid crystal is the most important parameters, that is, small  $K_{33}/K_{11}$  gives sharp threshold and hence high multiplexability. In this paper, a new method to determine  $K_{33}/K_{11}$  easily and correctly from capacitance-voltage curve is reported.

Almost all methods to determine  $K_{33}/K_{11}$  of dielectrically positive liquid crystal materials use the relation between retardation or capacitance of liquid crystal layer and applied electric or magnetic field.<sup>4-8</sup> However, in the case that the magnetic field is used, it is difficult to uniform the field and to adjust the direction accurately perpendicular to cell layer. The method using retardation is also disadvantageous because ordinary and extraordinary indices of refraction and cell thickness must be measured accurately beforehand. Therefore, the method using the electric field dependence of cell capacitance is best, because it has no disadvantages mentioned above. Especially the one using the initial slope of capacitance-voltage curve near threshold in a homogeneous cell<sup>5</sup> is most popular. This method is based on the following approximate relation between cell capacitance  $C$  and applied

voltage  $V$  near threshold voltage  $V_{th}$ ,

$$\frac{C - C_1}{C_1} \approx \frac{2 \cdot \gamma}{1 + \kappa + \gamma} \cdot \frac{V - V_{th}}{V_{th}} \quad (1)$$

In this equation,  $C_1$  is capacitance at  $V \leq V_{th}$ ,  $\gamma$  and  $\kappa$  are defined by

$$\gamma = \epsilon_{11} / \epsilon_1 - 1,$$

$$\kappa = K_{33} / K_{11} - 1,$$

where  $\epsilon_{11}$  and  $\epsilon_1$  are principle dielectric constants of liquid crystal. Eqn.1 is valid in only the neighborhood of threshold, so that the determination of elastic constants from eqn.1 includes noticeable error.<sup>8</sup>

Exact relation between cell capacitance  $C$  and applied voltage  $V$  are obtained by Gruler et al<sup>5</sup> as

$$\frac{V}{V_{th}} = \frac{2}{\pi} \cdot (1 + \gamma \sin^2 \phi_m)^{\frac{1}{2}} \cdot \int_0^{\phi_m} \left\{ \frac{1 + \kappa \sin^2 \phi}{(1 + \gamma \sin^2 \phi)(\sin^2 \phi_m - \sin^2 \phi)} \right\}^{\frac{1}{2}} d\phi, \quad (2)$$

$$\frac{C}{C_1} = \int_0^{\phi_m} \left\{ \frac{(1 + \kappa \sin^2 \phi)(1 + \gamma \sin^2 \phi)}{\sin^2 \phi_m - \sin^2 \phi} \right\}^{\frac{1}{2}} d\phi \bigg/ \int_0^{\phi_m} \left\{ \frac{1 + \kappa \sin^2 \phi}{(1 + \gamma \sin^2 \phi)(\sin^2 \phi_m - \sin^2 \phi)} \right\}^{\frac{1}{2}} d\phi, \quad (3)$$

where  $\phi$  is tilt angle of director of liquid crystal and  $\phi_m$  is tilt angle at the center of the cell.  $K_{33} / K_{11}$  can be determined by a numerical curve fitting method<sup>7</sup> using eqn.2 and 3, while the procedure of calculation is very complicated.

Then, we found another method to determine  $K_{33} / K_{11}$  from  $C$ - $V$  curve. The eqn.3 is transformed as

$$\frac{C}{C_1} = (1 + \gamma) - \frac{\gamma \cdot \int_0^{\phi_m} \left\{ \frac{1 + \kappa \sin^2 \phi}{(1 + \gamma \sin^2 \phi)(\sin^2 \phi_m - \sin^2 \phi)} \right\}^{\frac{1}{2}} \cos^2 \phi \cdot d\phi}{\int_0^{\phi_m} \left\{ \frac{1 + \kappa \sin^2 \phi}{(1 + \gamma \sin^2 \phi)(\sin^2 \phi_m - \sin^2 \phi)} \right\}^{\frac{1}{2}} d\phi}. \quad (4)$$

Eqn.2 and 4 give

$$\frac{C - C_1}{C_1} = \gamma - \frac{2 \cdot \gamma}{\pi} \cdot (1 + \gamma \sin^2 \phi_m)^{\frac{1}{2}} \cdot \frac{V_{th}}{V} \cdot \int_0^{\phi_m} \left\{ \frac{1 + \kappa \sin^2 \phi}{(1 + \gamma \sin^2 \phi)(\sin^2 \phi_m - \sin^2 \phi)} \right\}^{\frac{1}{2}} \cos^2 \phi \cdot d\phi \quad (5)$$

or

$$\frac{C - C_1}{C_1} = \gamma - \frac{2 \cdot \gamma}{\pi} \cdot (1 + \gamma \sin^2 \phi_m)^{\frac{1}{2}} \cdot \frac{V_{th}}{V} \int_0^{\sin \phi_m} \left\{ \frac{(1 + \kappa \cdot x^2) \cdot (1 - x^2)}{(1 + \gamma \cdot x^2) \cdot (\sin^2 \phi_m - x^2)} \right\}^{\frac{1}{2}} dx. \quad (6)$$

When applied voltage is sufficiently high, the approximation  $\phi_m \approx \pi/2$  is valid. Then eqn.6 is rewritten as

$$\frac{C-C_1}{C_1} = \gamma - \frac{2\gamma}{\pi} \cdot (1+\gamma)^{\frac{1}{2}} \cdot \frac{V_{th}}{V} \cdot \int_0^1 \left\{ \frac{1+\kappa \cdot x^2}{1+\gamma \cdot x^2} \right\}^{\frac{1}{2}} dx \quad (7)$$

or

$$C_R \equiv \frac{C-C_1}{C_{||}-C_1} = 1 - \frac{2}{\pi} \cdot (1+\gamma)^{\frac{1}{2}} \cdot \frac{V_{th}}{V} \cdot \int_0^1 \left\{ \frac{1+\kappa \cdot x^2}{1+\gamma \cdot x^2} \right\}^{\frac{1}{2}} dx, \quad (8)$$

where  $C_{||}$  is capacitance of homeotropic aligned cell. Calculated relations between  $C_R$  and  $V_{th}/V$  from eqn.2 and 3 are shown in Fig.1. Plots in this figure show the experimental characteristic of a cell with liquid crystal RO-TN605 (F.Hoffmann-La Roche). It is seen in this figure that linear variation of  $C_R$  and  $V_{th}/V$  are valid at  $C_R \approx 0.8$ . The slope  $\alpha$  is given as

$$\alpha = \frac{2}{\pi} \cdot (1+\gamma)^{\frac{1}{2}} \cdot \int_0^1 \left\{ \frac{1+\kappa \cdot x^2}{1+\gamma \cdot x^2} \right\}^{\frac{1}{2}} dx. \quad (9)$$

As is clear from eqn.7, the value of  $\gamma$  can be determined from extrapolation of  $(C-C_1)/C_1$  to  $1/V=0$ . Therefore,  $\kappa$  can be

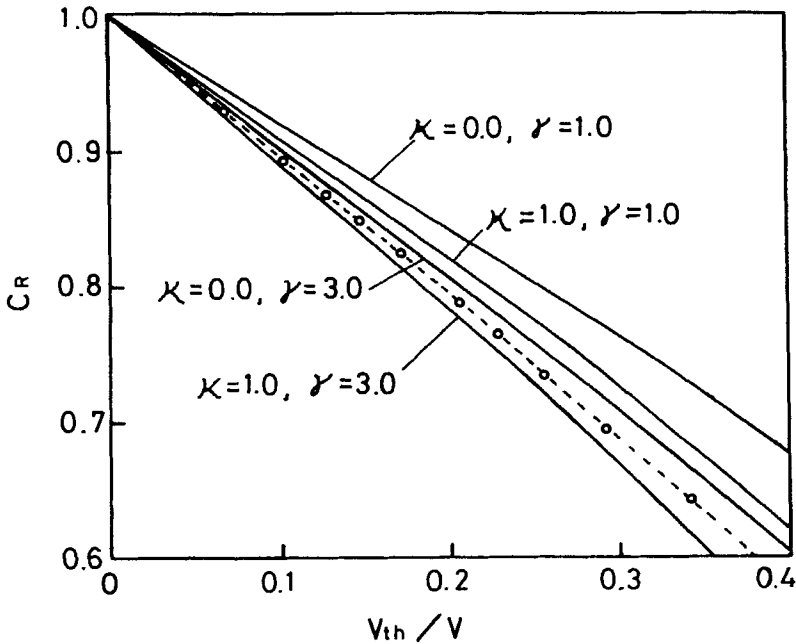


Figure 1. Relations between reduced capacitance  $C_R$  and  $V_{th}/V$ .

determined by eqn.7 with measured values of  $\alpha$  and  $\gamma$ . The relation between  $\alpha$  and  $\kappa$  with various values of  $\gamma$  are shown in Fig.2. It is seen that the relation between  $\alpha$  and  $\kappa$  or  $\gamma$  are relatively monotoneous, and therefore  $\kappa$  can be easily determined by interpolation of curves shown in Fig.2 instead of solving eqn.9.

In conclusion, the linear relation of capacitance with  $V_{th}/V$  at the high voltage is confirmed theoretically and experimentally. This linear relation exists over wide range  $C_R \approx 0.8-1.0$ , and the slope is function of  $K_{33}/K_{11}$  (or  $\kappa$ ) and  $\gamma$ . Therefore,  $K_{33}/K_{11}$  can be determined easily by the slope  $\alpha$  and  $\gamma$ .

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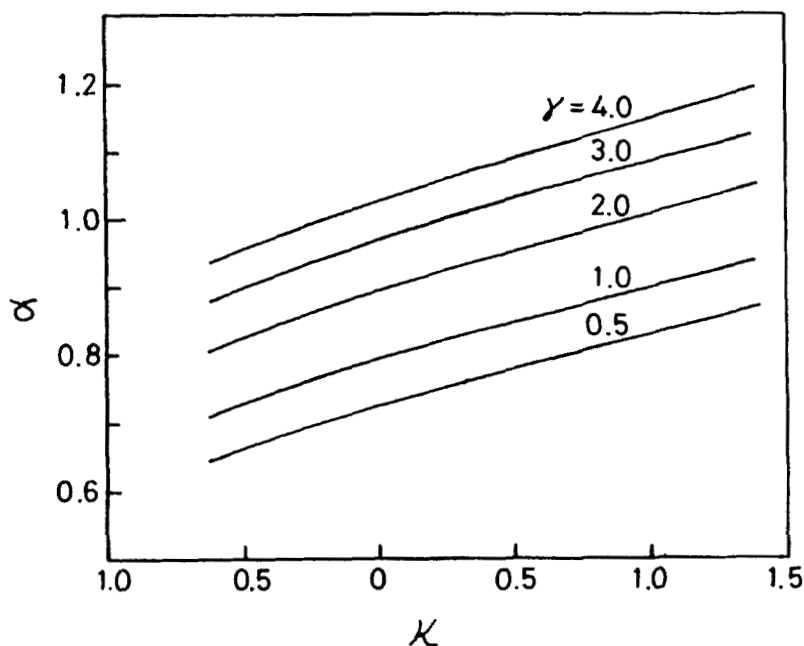


Figure 2. Relations between the slope  $\alpha$  and  $\kappa$  with various values of  $\gamma$ .

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